

Spin relaxation related to the edge scattering in graphene

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We discuss the role of spin-flip scattering of electrons from the magnetized edges in graphene nanoribbons. The spin-flip scattering is associated with strong fluctuations of the magnetic moments at the edge. Using the Boltzmann equation approach, which is valid for not too narrow nanoribbons, we calculate the spin relaxation time in the case of Berry-Mondragon and zigzag graphene edges. We also consider the case of ballistic nanoribbons characterized by very long momentum relaxation time in the bulk, when the main source of momentum and spin relaxation is the spin-dependent scattering at the edges. We found that in the case of zigzag edges, an anomalous spin diffusion is possible, which is related to very weak spin-flip scattering of electrons gliding along the nanoribbon edge.

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I. INTRODUCTION

One of the most challenging problems in the physics of graphene^{1,2} is related to the spin relaxation time of electrons or holes.^{3,4} This is really the key point for possible spintronic applications⁵ of this material.⁶ It was suggested before that the spin relaxation in graphene can be extremely weak due to very small intrinsic spin-orbit interaction.⁷⁻¹¹ The latter was extensively discussed in the past^{3,12-16} and, finally, quite persuasively confirmed by numerous investigations.^{4,17}

However, it is quite obvious that the spin relaxation rate can be essentially enhanced for graphene at a certain type of substrate generating strong Rashba spin-orbit coupling, or in graphene with some heavy adatoms¹⁸ and magnetic impurities¹⁹ or for graphene with short-wavelength surface ripples.^{7,20}

An additional mechanism of the spin relaxation can occur due to electron spin-flip scattering from the magnetized graphene edge. Such an effect of the magnetization of zigzag edges has been predicted by all first-principle as well as Hubbard-model calculations (for review, see Refs. 2,21), although experimental situation remains still not clear. Since the ordering of effectively one-dimensional system of magnetic moments at the edge is necessarily broken by fluctuations,²² the magnetized zigzag edge could be powerful source of the spin relaxation. However, as we demonstrate in this paper, this is not always the case because the scattering probability of electrons at the zigzag edge is vanishingly small for gliding electrons. Moreover, as we also show in this paper, in ballistic nanoribbons with magnetized zigzag edges,

an anomalous diffusion of the spin density is possible, mostly due to electrons gliding along the ribbon edge.²³

Our work is also motivated by the experiment,¹⁷ in which the enhanced spin relaxation time in hydrogenated graphene spin-valve devices has been found about 2 ns, so that the spin relaxation length was 7 μm .

II. MODEL

We consider the graphene ribbon of width L along the axis y , so that the graphene edges are at $x = 0$ and $x = L$ (see Fig. 1). We assume that the nanoribbon width L is not so small to take into consideration the size quantization, i.e., the wavelength of electrons at the Fermi level λ_F is much smaller than L . It enables using the semi-classical Boltzmann equation to describe the charge or spin transport. At the same time, we assume that L can be of the order or smaller than the electron mean free path ℓ . Therefore, in the following we consider in detail two main regimes: dirty nanoribbon when $\ell \ll L$, and ballistic nanoribbon with $\ell \gg L$.

Let us consider first the case of a dirty nanoribbon. The Boltzmann equation for the distribution function $f_{\mathbf{k}\sigma}(\mathbf{r}, t)$ of electrons with spin $\sigma = \uparrow, \downarrow$ is

$$\frac{\partial g_{\mathbf{k}\sigma}}{\partial t} + v_i \frac{\partial g_{\mathbf{k}\sigma}}{\partial r_i} = \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} (g_{\mathbf{k}'\sigma} - g_{\mathbf{k}\sigma}) + [\delta(x) + \delta(x - L)] a_0 \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} (g_{\mathbf{k}'\bar{\sigma}} - g_{\mathbf{k}\sigma}), \quad (1)$$

where $V_{\mathbf{k}\mathbf{k}'}$ is the probability of potential scattering of electrons in the bulk, $W_{\mathbf{k}\mathbf{k}'}$ is the probability of spin-flip

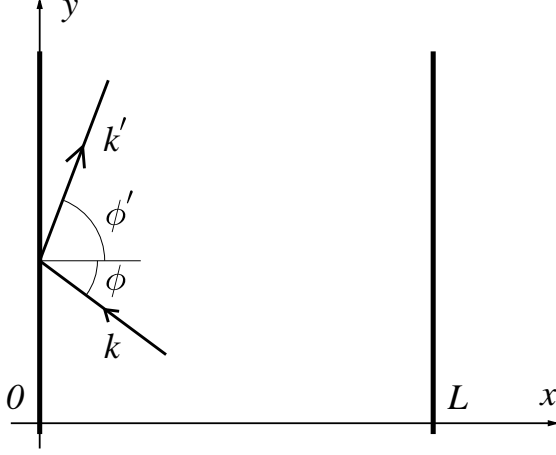


FIG. 1: Schematic presentation of the graphene ribbon along the axis y . Scattering of electrons from the edges at $x = 0$ and $x = L$ is associated with spin-flip transitions.

scattering at the edges $x = 0$ and $x = L$, $v_i = vk_i/k$, and we denote $f_{\mathbf{k}\sigma}(\mathbf{r}, t) = f_0(\varepsilon_{\mathbf{k}}) + g_{\mathbf{k}\sigma}(\mathbf{r}, t)$, where $f_0(\varepsilon_{\mathbf{k}})$ is the equilibrium Fermi-Dirac distribution function. In Eq. (1) we denoted by $\bar{\sigma}$ the opposite to σ state, and a_0 is the characteristic size of the transition region near the edge,²⁴ where the edge scatterers are located. In the following we take a_0 equal to the lattice constant of graphene.

We do not include intervalley scattering to kinetic equation (1) assuming that the scatterers are not so short ranged. For the spin-flip edge scattering, it means that we assume that the correlation length of magnetic fluctuations R_c is large with respect to the lattice constant of graphene, $R_c \gg a_0$.

The kinetic equation of this form (1) with delta-functions in the right hand part can be used to describe the dirty ribbon, $\ell \ll L$. In the opposite ballistic case of $\ell \gg L$, one should consider spin-flip scatterers as distributed within the ribbon (see below).

We can use Eq. (1) to describe the diffusion of spin density along the ribbon. Hence, we consider time evolution of the spin distribution along the ribbon, defined as

$$s_k(y, t) = \int_0^L dx \int \frac{d\Omega_{\mathbf{k}}}{2\pi} (g_{\mathbf{k}\uparrow} - g_{\mathbf{k}\downarrow}), \quad (2)$$

which includes averaging over orientation of the wavevector \mathbf{k} . Here we denoted by $\Omega_{\mathbf{k}}$ the polar angle of the vector \mathbf{k} .

If we choose the initial state with a smooth distribution $s_k(y, 0)$ along y -axis, which does not depend on x , then

$$s_k(y, t) = L \int \frac{d\Omega_{\mathbf{k}}}{2\pi} (g_{\mathbf{k}\uparrow} - g_{\mathbf{k}\downarrow}). \quad (3)$$

So, in the following we concentrate on the one-dimensional spin diffusion along the nanoribbon, which

is consistent with the experimental setup in Ref. 17.

For distances along the ribbon large with respect to ℓ we can use the diffusive approximation. Thus, we substitute $g_{\mathbf{k}\sigma}(y, t) = p_{k\sigma}(y, t) + v_y \xi_{k\sigma}(y, t)$, where the first term does not depend on direction of vector \mathbf{k} , and find the diffusion equation for $p_{k\sigma}$ in the bulk

$$\frac{\partial p_{k\sigma}}{\partial t} - D \frac{\partial^2 p_{k\sigma}}{\partial y^2} = 0, \quad (4)$$

and $\xi_{k\sigma} = -\tau (\partial p_{k\sigma} / \partial y)$, where $D = v^2 \tau$ is the one-dimensional diffusion coefficient, $\tau = \ell / v$, and $p_{k\sigma} = \int \frac{d\Omega_{\mathbf{k}}}{2\pi} g_{\mathbf{k}\sigma}$.

The general solution of Eq. (4) reads

$$p_{k\sigma}(y, t) = \int \frac{dq}{2\pi} B_{k\sigma}(q) e^{iqy - Dq^2 t}. \quad (5)$$

It gives us

$$g_{\mathbf{k}\sigma}(y, t) = \int \frac{dq}{2\pi} B_{k\sigma}(q) \left(1 - i\tau v q \frac{k_y}{k} \right) e^{iqy - Dq^2 t}. \quad (6)$$

where the coefficients $B_{k\sigma}(q)$ are related to the initial condition for the distribution $g_{\mathbf{k}\sigma}(y, 0)$.

Using Eqs. (2)-(4) one can also write the diffusion equation for spin density propagation in the bulk

$$\frac{\partial s_k}{\partial t} - D \frac{\partial^2 s_k}{\partial y^2} = 0, \quad (7)$$

which has the general solution

$$s_k(y, t) = L \int \frac{dq}{2\pi} [B_{k\uparrow}(q) - B_{k\downarrow}(q)] e^{iqy - Dq^2 t}. \quad (8)$$

The next step is to establish boundary conditions for the kinetic equations.

III. BOUNDARY CONDITION FOR THE DISTRIBUTION FUNCTION

Now we assume that the edges of the graphene nanoribbon are magnetized.^{2,21,25} It can be spontaneous magnetization as in the case of zigzag edge or due to magnetic impurities located at this edges.

The boundary condition at $x = 0$ is related to the spin-flip scattering from magnetic moments at the edge. It can be found by integrating Eq. (1) over a small region a_0 near the edge²⁴

$$|v_x| g_{\sigma}^>(k_y, 0) = |v_x| g_{\sigma}^<(k_y, 0) + a_0 \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} \times [g_{\sigma}^<(k'_y, 0) - g_{\sigma}^>(k_y, 0)]. \quad (9)$$

We consider first the case of ribbon with $L \gg \ell$. Then the distribution of incoming electrons over the angles is nearly homogeneous, $g_{\sigma}^<(k_y, 0) \simeq p_{\sigma}(0)$. Therefore, we can write Eq. (9) as

$$|v_x| g_{\sigma}^>(k_y, 0) = |v_x| p_{\sigma}(0) + W_{\mathbf{k}} [p_{\bar{\sigma}}(0) - g_{\sigma}^>(k_y, 0)], \quad (10)$$

where we denote $W_{\mathbf{k}} = a_0 \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'}$.

Similarly, we can obtain the corresponding boundary condition at $x = L$

$$|v_x| g_{\sigma}^<(k_y, L) = |v_x| p_{\sigma}(L) + W_{\mathbf{k}} [p_{\bar{\sigma}}(L) - g_{\sigma}^<(k_y, L)], \quad (11)$$

Using Eqs. (10) and (11) we find

$$g_{\sigma}^>(k_y, 0) = \frac{|v_x|}{|v_x| + W_{\mathbf{k}}} p_{\sigma}(0) + \frac{W_{\mathbf{k}}}{|v_x| + W_{\mathbf{k}}} p_{\bar{\sigma}}(0) \quad (12)$$

and

$$g_{\sigma}^<(k_y, L) = \frac{|v_x|}{|v_x| + W_{\mathbf{k}}} p_{\sigma}(L) + \frac{W_{\mathbf{k}}}{|v_x| + W_{\mathbf{k}}} p_{\bar{\sigma}}(L). \quad (13)$$

The spin flow through the left edge (spin drain) is

$$J_s(0) = \int \frac{d\Omega_{\mathbf{k}}}{\pi} |v_x| [p_{\uparrow}(0) - p_{\downarrow}(0) - g_{\uparrow}^>(k_y, 0) + g_{\downarrow}^>(k_y, 0)] = 2[p_{\uparrow}(0) - p_{\downarrow}(0)] \int \frac{d\Omega_{\mathbf{k}}}{\pi} \frac{|v_x| W_{\mathbf{k}}}{|v_x| + W_{\mathbf{k}}}, \quad (14)$$

whereas the linear spin density in the ribbon is (it does not depend on x)

$$s_k = L[p_{\uparrow}(0) - p_{\downarrow}(0)]. \quad (15)$$

The outgoing spin current (spin drain) through the right edge is the same, $J_s(L) = J_s(0)$.

Thus, we find the effective spin relaxation time

$$\frac{1}{\tau_s} = \frac{2J_s(0)}{s_k} = \frac{4}{\pi L} \int d\Omega_{\mathbf{k}} \frac{|v_x| W_{\mathbf{k}}}{|v_x| + W_{\mathbf{k}}}. \quad (16)$$

It corresponds to the relaxation of spin density in the nanoribbon. Note that, in fact, the functions $p_{k\sigma}$ do not depend on x , as discussed before.

IV. PROBABILITY OF SPIN-FLIP SCATTERING AT THE EDGE

As mentioned before, we assume that the main source of spin-flip scattering in the graphene nanoribbon is due to the magnetized graphene edge.^{2,21,25} Recently, this problem has been very intensively studied by different methods, and therefore the possibility of edge magnetization is generally recognized by researchers.

We consider the s - d exchange coupling Hamiltonian describing the coupling of electrons to the magnetic moments localized at the edge $x = 0$

$$H_{int} = W(x) \boldsymbol{\sigma} \cdot \mathbf{m}(y), \quad (17)$$

where $W(x)$ is a short-range exchange potential related to the shape of electron wavefunction in a state localized at the edge. Similar spin-flip scattering of electrons is at the other edge $x = L$.

Let us assume that the distribution of the moments $\mathbf{m}(y)$ along the edge $x = 0$ has the following form

$$\mathbf{m}(y) = \mathbf{m}_0 + \delta\mathbf{m}(y), \quad (18)$$

where \mathbf{m}_0 is a constant part of magnetization (one can take it oriented along the quantization axis z) and $\delta\mathbf{m}(y)$ is a random fluctuation part, such that $\langle \delta\mathbf{m}(y) \rangle = 0$, $\langle \delta m_{\alpha}(y) \delta m_{\beta}(y') \rangle = \gamma \delta_{\alpha\beta} e^{-\lambda|y-y'|}$ (here $\alpha, \beta = x, y$), and $\lambda^{-1} \equiv R_c$ is the correlation length of magnetic fluctuations.²² The constant γ characterizes the amplitude of fluctuations, $\gamma = \langle (\delta\mathbf{m})^2 \rangle$.

The matrix element of spin-flip interaction (17) of electrons with the fluctuating magnetic moments at the edge (here $\psi_{\mathbf{k}}$ is spinor in the sublattice space of graphene)

$$w_{\mathbf{k}\mathbf{k}'} = \int d^2\mathbf{r} \psi_{\mathbf{k}}^{\dagger}(\mathbf{r}) W(x) [\delta m_x(y) - i \delta m_y(y)] \psi_{\mathbf{k}'}(\mathbf{r}), \quad (19)$$

and, after averaging over fluctuating moments, for the corresponding spin-flip probability $W_{\mathbf{k}\mathbf{k}'} = \frac{2\pi}{\hbar} \langle |w_{\mathbf{k}\mathbf{k}'}|^2 \rangle \delta(\varepsilon_k - \varepsilon_{k'})$ we obtain

$$W_{\mathbf{k}\mathbf{k}'} = \frac{4\pi\gamma}{\hbar} \int d^2\mathbf{r} d^2\mathbf{r}' e^{-\lambda|y-y'|} \psi_{\mathbf{k}}^{\dagger}(\mathbf{r}) W(x) \psi_{\mathbf{k}'}(\mathbf{r}) \times \psi_{\mathbf{k}'}^{\dagger}(\mathbf{r}') W(x') \psi_{\mathbf{k}}(\mathbf{r}') \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}). \quad (20)$$

To calculate the integrals in Eq. (20) we need to know the wavefunctions $\psi_{\mathbf{k}}(\mathbf{r})$ near the graphene edge. It is well known that the choice of these functions depends on the type of the edge, and can be described by using different boundary conditions for the wave functions.

A. Berry-Mondragon boundary condition for the wavefunction at the edge

Strictly speaking, in the case of Berry-Mondragon (BM) boundary we probably should not expect any spontaneous magnetization of the graphene edge. Most probably, this effect is associated only with the zigzag-shaped^{22,26} and chiral²⁷ edges. Nevertheless, in the absence of any firm confirmation of these mostly theoretical suggestions, we consider the effect of magnetization for different types of the boundaries. Note that the magnetized boundary can be also achieved due to correlated magnetic atoms intentionally inserted at the graphene edges.

The Berry-Mondragon type of the boundary conditions²⁸ relates the pseudo-spinor components of the wavefunction in graphene to those in the vacuum, described by a large gap in the graphene Hamiltonian. In this case the wave function near $x = 0$ acquires to following form (here \mathcal{L} is the ribbon length)

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{2\mathcal{L}}} \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad (21)$$

and from Eq. (20) we then obtain

$$W_{\mathbf{k}\mathbf{k}'}^{(bm)} = \frac{\pi\gamma W_0^2}{\hbar\mathcal{L}L^2} \int dy e^{-\lambda|y|} e^{-i(k_y - k'_y)y} \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}), \quad (22)$$

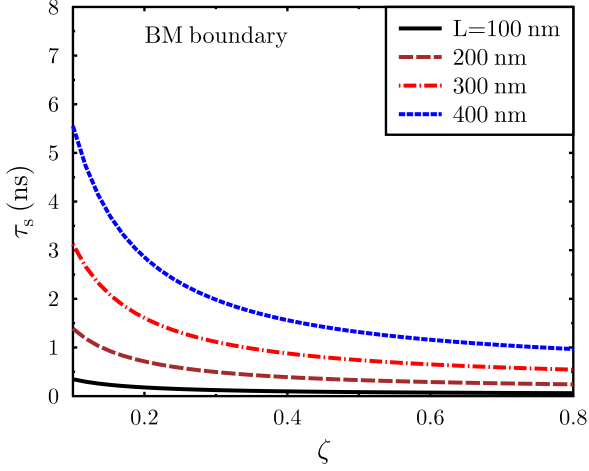


FIG. 2: Dependence of effective spin relaxation time on the parameter ζ for different values of the ribbon width. The case of BM boundary.

where $W_0 = \int dx e^{i(k-k')x} W(x)$ is a constant, provided the potential $W(x)$ is short ranged.

After calculating the integral over y in Eq. (22) we come to

$$W_{\mathbf{k}\mathbf{k}'}^{(bm)} = \frac{2\pi\gamma W_0^2 \lambda \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})}{\hbar \mathcal{L} L^2 [\lambda^2 + (k_y - k'_y)^2]}. \quad (23)$$

Then, integrating (23) over \mathbf{k}' we find the function $W_{\mathbf{k}}$, which determines the spin relaxation time in Eq. (16)

$$W_{\mathbf{k}}^{(bm)} = \frac{2\gamma W_0^2 a_0 \zeta}{\hbar^2 L v} \int_{-\pi/2}^{\pi/2} \frac{d\phi'}{1 + \zeta^2 (\sin \phi - \sin \phi')^2}, \quad (24)$$

where we denote the parameter $\zeta = kR_c$ and introduce the angles ϕ, ϕ' of, respectively, incoming and outgoing electrons by $k_y = k \sin \phi$ and $k'_y = k \sin \phi'$ (see Fig. 1). Of course, the value $k = k_F$ is relevant for the electronic transport.

The results of numerical calculations with (16) and (24) are presented in Fig. 2, where we used the following parameters: $v = 10^8$ cm/s, $\gamma = 1$, $a_o = 5 \times 10^{-8}$ cm, $W_0/a_0 = 0.2$ eV.

B. Zigzag boundary

In the case of zigzag boundary of graphene, the possibility of edge magnetization has been established in several works (see, e.g., the review article of Yazyev²¹ and Chap. 12 of Ref. 2).

For the zigzag boundary we can use the wavefunction near the edge $x = 0$ in the following form²⁹

$$\psi_{\mathbf{k}}(\mathbf{r}) \simeq \frac{e^{ik_y y}}{\sqrt{\mathcal{L}L}} \begin{pmatrix} \sin k_x x \\ -\frac{ik_x}{k} \cos k_x x + \frac{ik_y}{k} \sin k_x x \end{pmatrix}. \quad (25)$$

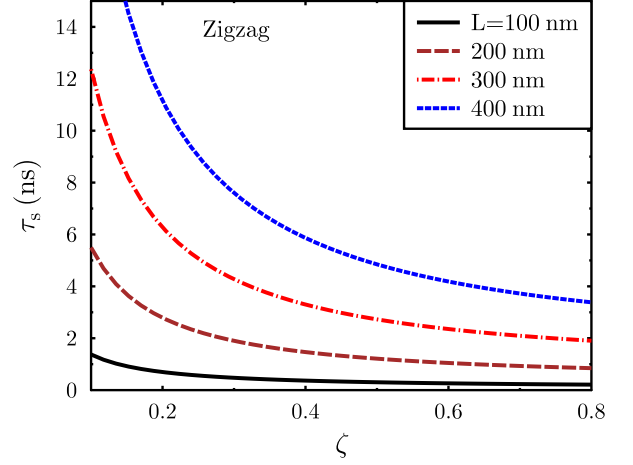


FIG. 3: The same as in Fig. 2 for the case of zigzag boundary.

This type of the boundary condition corresponds to requirement of zero value for one of the spinor components, and nonzero value for another one.

Substituting (25) in Eq. (20) we obtain

$$W_{\mathbf{k}\mathbf{k}'}^{(z)} = \frac{\pi\gamma W_0^2 k_x^2 k'^2}{\hbar \mathcal{L} L^2 k^4} \int dy e^{-\lambda|y|} e^{-i(k_y - k'_y)y} \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}), \quad (26)$$

and, correspondingly, we get

$$W_{\mathbf{k}\mathbf{k}'}^{(z)} = \frac{2\pi\gamma W_0^2 k_x^2 k'^2 \lambda \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})}{\hbar \mathcal{L} L^2 k^4 [\lambda^2 + (k_y - k'_y)^2]}. \quad (27)$$

Now we can also calculate the function $W_{\mathbf{k}}^{(z)}$

$$W_{\mathbf{k}}^{(z)} = \frac{2\gamma W_0^2 a_0 \zeta \cos^2 \phi}{\hbar^2 L v} \int_{-\pi/2}^{\pi/2} \frac{\cos^2 \phi' d\phi'}{1 + \zeta^2 (\sin \phi - \sin \phi')^2}. \quad (28)$$

Then using Eqs. (16) and (28) we calculated numerically the spin relaxation time for the case of nanoribbon with zigzag edges in the regime of $\ell \ll L$. The dependence of $\tau_s(\zeta)$ for the same parameters as in Fig. 1 are presented in Fig. 3.

V. ANOMALOUS SPIN DIFFUSION IN THE BALLISTIC NANORIBBON

In the ballistic regime of $L \ll \ell$ we can completely neglect the dependence of the distribution function on x in Eq. (1), and consider the spin-flip scattering as scattering within the bulk. It should be stressed that this condition of $L \ll \ell$ is, in fact, limiting only the value of L , whereas the mean free path is still finite (and in principle can be not so large) which justifies using the diffusion approach (see below).

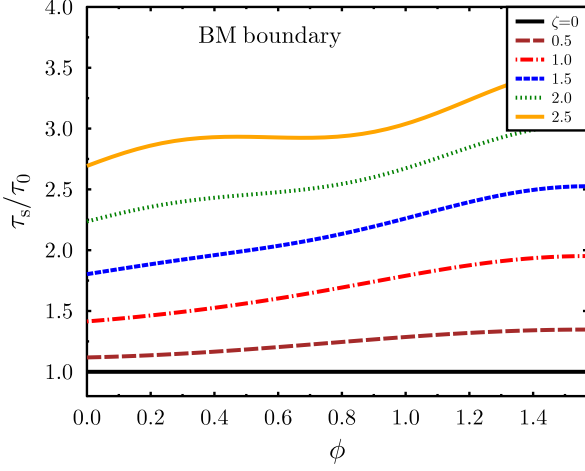


FIG. 4: The dependence of spin relaxation time on the angle ϕ in the ballistic regime with BM boundaries.

Then, by using the diffusive approximation for the y -dependence of spin-resolved distribution, one can obtain

$$\frac{\partial p_{k\sigma}}{\partial t} - D \frac{\partial^2 p_{k\sigma}}{\partial y^2} = \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} (p_{k\bar{\sigma}} - p_{k\sigma}), \quad (29)$$

For the case of BM edge we substitute (23) into Eq. (29)

$$\begin{aligned} \frac{\partial p_{k\sigma}}{\partial t} - D \frac{\partial^2 p_{k\sigma}}{\partial y^2} &= \frac{2\pi\gamma W_0^2 \lambda (p_{k\bar{\sigma}} - p_{k\sigma})}{\hbar \mathcal{L} L^2} \\ &\times \sum_{\mathbf{k}'} \frac{\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})}{\lambda^2 + (k_y - k'_y)^2} \end{aligned} \quad (30)$$

and find the diffusion equation for spin density

$$\frac{\partial s_k}{\partial t} - D \frac{\partial^2 s_k}{\partial y^2} = -\frac{s_k}{\tau_s}, \quad (31)$$

where

$$\frac{1}{\tau_s} = \frac{2\gamma W_0^2 \zeta}{\hbar^2 L v} \int_{-\pi/2}^{\pi/2} \frac{d\phi'}{1 + \zeta^2 (\sin \phi - \sin \phi')^2}. \quad (32)$$

The dependence of τ_s/τ_0 on the angle ϕ is presented in Fig. 4, where we denoted $\tau_0^{-1} = 2\pi\gamma W_0^2 \zeta / \hbar^2 L v$. As we see, the spin relaxation time is anisotropic (i.e., depending on the angle ϕ of incoming electrons) but this anisotropy is relatively weak. It justifies the diffusive equation (31) for ballistic nanoribbon with BM boundaries.

If we try to do the same for the ballistic ribbon with zigzag edges, then after using (26) we come to the following result

$$\frac{1}{\tau_s} = \frac{2\gamma W_0^2 \zeta \cos^2 \phi}{\hbar^2 L v} \int_{-\pi/2}^{\pi/2} \frac{\cos^2 \phi' d\phi'}{1 + \zeta^2 (\sin \phi - \sin \phi')^2} \quad (33)$$

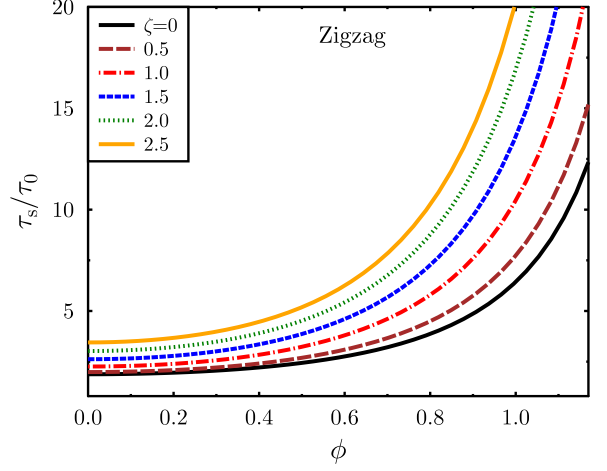


FIG. 5: The dependence of spin relaxation time on the angle ϕ in the ballistic regime with zigzag boundaries.

The corresponding dependence of τ_s on the angle ϕ is presented in Fig. 5. In this case the anisotropy of spin relaxation time is very strong.

Strictly speaking, the diffusion equation (45) cannot be used to describe the spin propagation for the ribbon with zigzag edges because gliding electrons can propagate at a long distance without changing their spin. Correspondingly, we anticipate that the ballistic transport of charge in such a ribbon is accompanied by the ballistic spin transport.

Anomalous spin diffusion in the zigzag nanoribbon

In the case of ballistic zigzag nanoribbon, by using the Boltzmann equation for $g_{\mathbf{k}\sigma}$ we can find the kinetic equation for $s_{\mathbf{k}} \equiv g_{\mathbf{k}\uparrow} - g_{\mathbf{k}\downarrow}$ in the following form

$$\begin{aligned} \frac{\partial s(\phi)}{\partial t} + v \sin \phi \frac{\partial s(\phi)}{\partial y} &= -\frac{\cos^2 \phi}{\tau_0} \\ &\times \int_{-\pi/2}^{\pi/2} \frac{\cos^2 \phi' [s(\phi) + s(\phi')]}{1 + \zeta^2 (\sin \phi - \sin \phi')^2} d\phi', \end{aligned} \quad (34)$$

where $s(\phi)$ explicitly shows the dependence of spin distribution on the orientation of vector \mathbf{k} .

Let us consider the limiting cases of short and long correlation lengths R_c in comparison with electron wavelength λ_F .

1. Large magnetic correlation length, $\zeta \gg 1$

In the case of $\zeta \gg 1$ (the correlation length is much larger than the electron wavelength at the Fermi level

λ_F), the main contribution in intergral (34) is from $\phi' \simeq \phi$. Then we can simplify Eq. (34) to

$$\frac{\partial s(\phi)}{\partial t} + v \sin \phi \frac{\partial s(\phi)}{\partial y} = -\frac{2C \cos^3 \phi s(\phi)}{\tau_0}, \quad (35)$$

where we denote

$$C = \frac{1}{\cos \phi} \int_{-\pi/2}^{\pi/2} \frac{\cos^2 \phi' d\phi'}{1 + \zeta^2 (\sin \phi - \sin \phi')^2} \simeq \frac{\pi}{\zeta}. \quad (36)$$

After Fourier transformation over y , Eq. (35) reads

$$\frac{\partial s_q(\phi)}{\partial t} + ivq \sin \phi s_q(\phi) = -\frac{2\pi \cos^3 \phi s_q(\phi)}{\tau_0 \zeta}. \quad (37)$$

This equation has the solution

$$s(\phi) = \int \frac{dq}{2\pi} R_q \exp \left(i q y - i v q t \sin \phi - \frac{2\pi t \cos^3 \phi}{\tau_0 \zeta} \right), \quad (38)$$

where the arbitrary function R_q should be related to the initial condition. We choose $R_q = 1$, which corresponds to the δ -function spin distribution at $t = 0$.

Now we integrate (38) over angle ϕ . It gives us the spin distribution at the moment t

$$s(t) = \frac{1}{\sqrt{v^2 t^2 - y^2}} \exp \left[-\frac{2\pi t}{\tau_0 \zeta} \left(1 - \frac{y^2}{v^2 t^2} \right)^{3/2} \right], \quad (39)$$

where $|y| < vt$.

Then we find

$$\begin{aligned} \langle y^2(t) \rangle &\equiv \int_{-vt}^{vt} dy y^2 s(t) = v^2 t^2 \int_{-1}^1 \frac{z^2 dz}{\sqrt{1-z^2}} \\ &\times \exp \left[-\frac{2\pi t}{\tau_0 \zeta} (1-z^2)^{3/2} \right], \end{aligned} \quad (40)$$

where $z = y/vt$. The dependence on t of the integral in Eq. (40) describes deviation from pure ballistics. As follows from Eq. (40), for $t \ll \tau_0 \zeta$ the spin transport is ballistic.

At large times, $t \gg \tau_0 \zeta$, we obtain from (40)

$$\langle y^2(t) \rangle \simeq \frac{2v^2 t^2}{3} \int_0^1 \frac{x^{-2/3} dx}{\sqrt{1-x^{2/3}}} e^{-2\pi t x / \tau_0 \zeta} \quad (41)$$

$$\simeq \frac{2v^2 (\tau_0 \zeta)^{1/3} t^{5/3} \Gamma(\frac{4}{3})}{(2\pi)^{1/3}} \quad (42)$$

It corresponds to anomalous spin diffusion with characteristic diffusion length $l_d \equiv \sqrt{\langle y^2(t) \rangle} \sim t^{5/6}$.

2. Small correlation length, $\zeta \ll 1$

Now we consider the case of small magnetic correlation length, i.e., $\zeta \ll 1$. In this case we obtain from (34) (note

that in the case of small ζ , the characteristic relaxation time τ_0 is large since $\tau_0 \sim \zeta^{-1}$)

$$\begin{aligned} \frac{\partial s(\phi)}{\partial t} + v \sin \phi \frac{\partial s(\phi)}{\partial y} &= -\frac{\pi \cos^2 \phi s(\phi)}{2\tau_0} \\ &- \frac{\cos^2 \phi}{\tau_0} \int_{-\pi/2}^{\pi/2} \cos^2 \phi' s(\phi') d\phi'. \end{aligned} \quad (43)$$

The solution of Eq. (43) is

$$\begin{aligned} s(\phi, t, y) &= \delta(y - vt \sin \phi) e^{-\pi t \cos^2 \phi / 2\tau_0} - \frac{\cos^2 \phi}{\tau_0} \\ &\times \int_{y-vt}^{y+vt} dy' \int_0^{t-|y-y'|/v} dt' \delta[y - y' - v(t-t') \sin \phi] \\ &\times e^{-\pi(t-t') \cos^2 \phi / 2\tau_0} A(t', y'), \end{aligned} \quad (44)$$

where we denoted

$$A(t, y) = \int_{-\pi/2}^{\pi/2} \cos^2 \phi s(\phi, t, y) d\phi. \quad (45)$$

After multiplying (44) by $\cos^2 \phi$ and integrating over ϕ one can obtain the integral equation for the function $A(t, y)$

$$\begin{aligned} A(t, y) &= \frac{\sqrt{v^2 t^2 - y^2}}{v^2 t^2} e^{-\pi t(1-y^2/v^2 t^2)/2\tau_0} \theta(vt - |y|) \\ &- \frac{1}{\tau_0} \int_{y-vt}^{y+vt} dy' \int_0^{t-|y-y'|/v} dt' \frac{[v^2(t-t')^2 - (y-y')^2]^{3/2}}{v^4(t-t')^4} \\ &\times e^{-\pi(t-t')[1-(y-y')^2/v^2(t-t')^2]/2\tau_0} A(t', y'). \end{aligned} \quad (46)$$

The integrated over angles spin distribution can be found from Eq. (44)

$$\begin{aligned} s(t, y) &= \frac{e^{-\pi t(1-y^2/v^2 t^2)/2\tau_0}}{\sqrt{v^2 t^2 - y^2}} \theta(vt - |y|) \\ &- \frac{1}{\tau_0} \int_{y-vt}^{y+vt} dy' \int_0^{t-|y-y'|/v} dt' \frac{\sqrt{v^2(t-t')^2 - (y-y')^2}}{v^2(t-t')^2} \\ &\times e^{-\pi t[1-(y-y')^2/v^2(t-t')^2]/2\tau_0} A(t', y'). \end{aligned} \quad (47)$$

Using spin distribution (47) we can also find the mean square distance, at which the spin density propagates in time t

$$\begin{aligned} \langle y^2(t) \rangle &= v^2 t^2 \int_{-1}^1 \frac{z^2 e^{-\pi t(1-z^2)/2\tau_0} dz}{\sqrt{1-z^2}} - \frac{v^3 t^4}{\tau_1} \int_{-\infty}^{\infty} z^2 dz \\ &\times \int_{z-1}^{z+1} dz' \int_0^{1-|z-z'|} d\xi \frac{\sqrt{(1-\xi)^2 - (z-z')^2}}{(1-\xi)^2} \\ &\times e^{-\pi t(1-\xi)[1-(z-z')^2]/2\tau_0} A(t\xi, z'), \end{aligned} \quad (48)$$

where we denoted $z = y/vt$, $z' = y'/vt$, and $\xi = t'/t$. If $t \ll \tau_0$ then the spin propagation is pure ballistic at a large distance, $l \simeq v\tau_0$.

If $t \gg \tau_0$, the main contribution to the first integral in (48) is from $z \simeq 1$, and in the second integral from $\xi \sim 1$ and $|z - z'| \ll 1$. Then we obtain from (48)

$$\langle y^2(t) \rangle \simeq 2^{-1/2} v^2 t^{3/2} \tau_0^{1/2} - \frac{v^3 t^4}{\tau_0} \int_{-\infty}^{\infty} z^2 dz \int_0^1 dr \int_0^{1-r} d\xi \times \frac{[A(t\xi, z+r) + A(t\xi, z-r)]}{1-\xi} e^{-\pi t(1-\xi)(1-r)/\tau_0}. \quad (49)$$

The function $A(t, y)$ is calculated in Appendix. Since $A(t, z)$ exponentially decays at $t \gg \tau_0$, the main contribution in the last integral of (49) is from the vicinity of $r \sim 1$ and $\xi \ll 1$. Correspondingly, we get

$$\langle y^2(t) \rangle \simeq 2^{-1/2} v^2 t^{3/2} \tau_0^{1/2} - \frac{v^3 t^4}{\tau_0} \int_{-\infty}^{\infty} z^2 dz \times \int_0^1 dx \int_0^x d\xi A(t\xi, z \pm 1) e^{-\pi t x / \tau_0}. \quad (50)$$

Here the main contribution to the integral comes from $x \sim \xi \sim \tau_0/t \ll 1$. Then using (A10) we finally obtain

$$\langle y^2(t) \rangle \simeq \left(\frac{1}{\sqrt{2}} - \frac{2}{\pi^2} \right) v^2 t^{3/2} \tau_0^{1/2}, \quad (51)$$

which leads to the anomalous spin diffusion law with the diffusion length $l_d \sim t^{3/4}$.

VI. CONCLUSION

We studied the effect of spin-flip scattering of electrons from the magnetized edges of graphene nanoribbons. The essential point of our model is an assumption of strong fluctuations of the magnetic moments at the graphene edge, which have been established earlier in Ref. 22. The spin-flip scattering of this type, which is relevant for spin relaxation in graphene nanoribbons, can be strongly suppressed for electron incoming under small angles to the edge. We found that this effect is especially strong for the zigzag boundary of graphene. For such gliding electrons the spin is nearly conserved. As a result, there is a possibility of anomalous spin diffusion along the graphene nanoribbon.

For the estimation of parameters we can use the relation for conductivity in graphene $\sigma = e^2 k_F \ell / \pi \hbar$, which gives us for the mobility $\mu = e \tau v / \hbar k_F$. For the carrier density in graphene we use $n = k_F^2 / \pi$. We can assume the mean free path of electrons in the bulk $\ell = 3 \times 10^{-4}$ cm and $n = 10^{12}$ cm $^{-2}$. It gives us the bulk relaxation time $\tau = \ell / v = 3 \times 10^{-12}$ s, $k_F \simeq 1.8 \times 10^6$ cm $^{-1}$ and $\mu \simeq 2.5 \times 10^5$ cm 2 /V·s, which can be achieved in graphene.³⁰ This gives us the estimation of characteristic width for the ribbon, that is, the ballistic case corresponds to $L \ll 10^{-4}$ cm.

The correlation length R_c has been calculated in Ref. 22 as a function of temperature. According to this estimation it can vary from 1000 nm at very low temperatures to about 1 nm at the room temperature. Thus, for

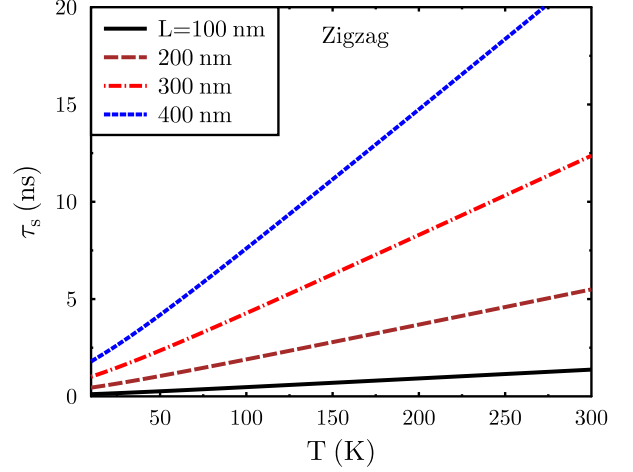


FIG. 6: Dependence of spin relaxation time on temperature for different values of the ribbon width. The case of zigzag boundary, and we take $k_F = 1 \times 10^6$ cm $^{-1}$.

the presented above parameters of graphene (i.e., not too small density of electrons) we can expect the value of parameter $\zeta = k_F R_c \ll 1$. In our approach it corresponds to the case of small correlation length.

It should be noted that the temperature dependence of correlation length R_c determines the temperature dependence of spin relaxation in our model. In temperature range of 10 to 100 K, one can approximate it²² by $R_c \simeq 100/k_B T$ (nm). Then according to Eqs. (16),(28) the relaxation time grows with the temperature. Figure 6 demonstrates that this dependence is almost linear. At high temperatures, when the correlation length is much smaller than the electron wave length, electrons do not feel fluctuating spins because the magnetic disorder is effectively averaged over the wave length.

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Appendix A: Calculation of the function $A(t, y)$

Using the Fourier transformation of Eq. (43)

$$\left(-i\omega + ivq \sin \phi + \frac{\pi \cos^2 \phi}{2\tau_0}\right) s_{\omega q} = -\frac{\cos^2 \phi A_{\omega q}}{\tau_0} \quad (\text{A1})$$

we find

$$s_{\omega q} = -\frac{\cos^2 \phi A_{\omega q}}{-i\omega\tau_0 + ivq\tau_0 \sin \phi + \frac{\pi}{2} \cos^2 \phi}. \quad (\text{A2})$$

Then after multiplying (A2) by $\cos^2 \phi$ and integrating over angle ϕ we come to

$$A_{\omega q} \left(1 + \int_{-\pi/2}^{\pi/2} \frac{\cos^4 \phi d\phi}{-i\omega\tau_0 + ivq\tau_0 \sin \phi + \frac{\pi}{2} \cos^2 \phi}\right) = 0. \quad (\text{A3})$$

From (A3) the condition of nonzero $A_{\omega q}$ gives us the equation

$$1 + \int_{-\pi/2}^{\pi/2} \frac{\cos^4 \phi d\phi}{-i\omega\tau_0 + ivq\tau_0 \sin \phi + \frac{\pi}{2} \cos^2 \phi} = 0 \quad (\text{A4})$$

determining the dependence $\omega(q)$.

Thus, the function $A(t, y)$ can be presented in the form

$$A(t, y) = \int \frac{dq}{2\pi} e^{iqy - i\omega(q)t}, \quad (\text{A5})$$

where we choose $A_{\omega q} = 1$, which corresponds to the assumed initial spin distribution $s(t = 0, y)$.

Let us look for the solution of Eq. (A4) in the form

$$\omega(q) = -ix(q)/\tau_0 \quad (\text{A6})$$

with $x(q)$ real. Then for $t \gg \tau_0$ the integral over q in (A5) is mostly determined by q near the minimum of dependence $x(q)$.

Substituting (A6) to (A4) we obtain

$$\int_{-\pi/2}^{\pi/2} \frac{\cos^4 \phi d\phi}{x - i\kappa \sin \phi - \frac{\pi}{2} \cos^2 \phi} = 1 \quad (\text{A7})$$

where we denoted $\kappa = vq\tau_0$.

The dependence $x(\kappa)$ calculated numerically from (A7) is shown in Fig. 7. It can be interpreted as the diffusion mode of a partial spin density related to electrons moving in transversal direction. Such electrons are strongly scattered from the edges, and therefore such mode is exponentially decaying (it has a gap). In the vicinity of

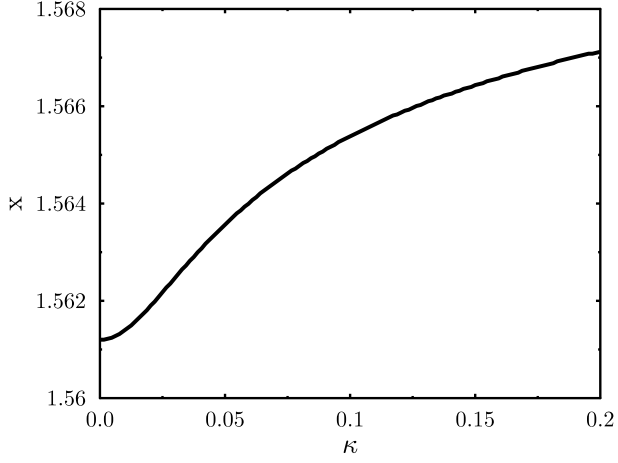


FIG. 7: The dependence $x(\kappa)$ calculated numerically from Eq. (A7).

$\kappa = 0$, it can be approximated by $x(\kappa) \simeq \beta + \alpha\kappa^2$, with $\beta \simeq 1.561$. Indeed, here the characteristic values are $y \sim vt$, $q \sim 1/y \sim 1/vt$, so that $\kappa \sim \tau_0/t \ll 1$, which corresponds to very close vicinity of the minimum in Fig. 7.

Substituting this approximation to (A5) we find

$$A(t, y) = \frac{e^{-\beta t/\tau_1}}{vt^{1/2}\tau_1^{1/2}} \psi(\tilde{y}), \quad (\text{A8})$$

where $\tilde{q} = v\sqrt{t\tau_0} q$, $\tilde{y} = y/v\sqrt{t\tau_0}$ and

$$\psi(\tilde{y}) = \frac{e^{-\tilde{y}^2/4\alpha}}{2\sqrt{\pi\alpha}} \quad (\text{A9})$$

is a universal function which does not depend on any parameters.

Substituting (A9) to the integral for $A(\tau_0, z)$ and using relation $z = y/vt = (\tau_0/t)^{1/2}\tilde{y}$, we obtain

$$\int z^2 dz A(\tau_0, z \pm 1) \simeq \frac{2}{v\sqrt{t\tau_0}}. \quad (\text{A10})$$